

Now construct the graph  $G'''$  by adding the point  $x'''$  as in Figure 3 to the points corresponding to  $y$  and  $z$ .  $p(x''') = p(x) = e$ . In  $C''$ , the parities of  $y''$  and  $z''$  are reversed from the parities of the corresponding  $y$  and  $z$  in  $C$ , whence by adding the lines  $(y''', x''')$   $(x''', z''')$  these parities are restored. Thus  $P(C''') = P(C)$ , and the theorem holds.

CASE 2.  $n + 1$  even. If  $p(x) = 0$  for every  $x \in C$ ,  $G'$  consists of the simple cycle  $C'$  with the same number of points as  $C$ , with an additional point connected to each point of  $C'$ .

If  $p(x) \neq 0$  for some  $x$ , proof as in Case 1.

#### REFERENCE

1. B. GRUNBAUM, *Convex Polytopes* (to be published).

NORMAN C. DALKEY  
The RAND Corporation  
Santa Monica, California

## A Characterization of Planar Geodetic Graphs\*

An undirected graph is *geodetic* if each pair of vertices is joined by a unique shortest arc (path), called a *geodesic*. The problem of characterizing the geodetic graphs has been posed by O. Ore [1]. The solution for planar graphs is announced.

Clearly a graph is geodetic if and only if each of its blocks is a geodetic subgraph. Let  $K_n$  denote the complete graph with  $n$  vertices. A *suspended arc* is an arc whose terminal vertices are of degree at least 3 while any intermediate vertices are of degree 2.

**THEOREM.** *A planar graph is geodetic if and only if it is connected and each of its blocks is  $K_2$ , an odd cycle, or a homeomorph of  $K_4$  which satisfies the following three conditions:*

---

\* The result in this note is contained in the author's dissertation, presented for the degree of Doctor of Philosophy in Yale University. This research was partially supported by National Science Foundation Grant No. 18895.

1. *Each cycle which is the union of three suspended arcs has odd length.*
2. *The three cycles which are unions of four suspended arcs have the same even length.*
3. *Each suspended arc is a geodesic.*

The present proof is extremely long and is omitted. An example of a homeomorph of  $K_4$  satisfying the above conditions is shown in Fig. 1.

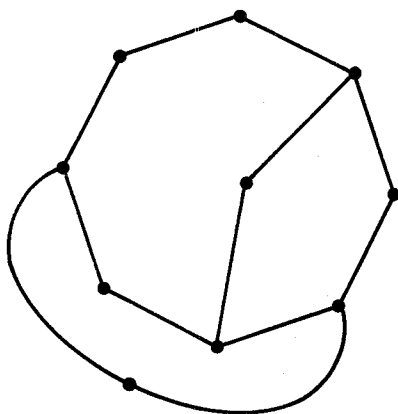


Fig. 1.

#### REFERENCE

1. O. ORE, *Theory of Graphs*, American Mathematical Society, Providence, Rhode Island, 1962.

MARK E. WATKINS  
*University of North Carolina*  
*Chapel Hill, North Carolina*